International Journal of Fluid Mechanics Research, Vol. **, No. *, 200*

Effects of Thermal Radiation on Natural Convection in a Porous Medium†

M. F. El-Amin[1], **Ibrahim Abbas**[2], and **Rama Subba Reddy Gorla**[3]

[1]Mathematics Department, Aswan Faculty of Science, South Valley University
Aswan 81258, Egypt

[2]Mathematics Department, Sohag Faculty of Science, South Valley University
Sohag, Egypt

[3]Department of Mechanical Engineering, Cleveland State University
Cleveland, OH 44115, USA

An analysis is presented for the thermal radiation effect on the non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium. Forchheimer extension is considered in the flow equations. Rosseland approximation is used to describe the radiative heat flux in the energy equation. The nondimensional governing partial equations are solved by the finite element method. The resulting nonlinear integral equations are linearized and solved by the Newton-Raphson iteration. Results for the details of the stream function, velocity. and temperature contours and profiles. as well as heat transfer rate in terms of Nusselt number. are shown graphically.

* * *

Nomenclature

A	constant;
C	empirical constant;
C_T	temperature difference;
d	pore diameter;
g	gravitatoinal constant;
K	permeability of the porous medium;
k	thermal conductivity;
Nu_x	local Nusselt number;
p	pressure;
q	local heat flux;
q^r	radiative heat flux;

†Received 16.02.2006

ISSN 1064-2277

R	radiation parameter;
Ra	Rayleigh number;
$\bar{T}$	temperature;
T	nondimensional temperature;
$\bar{u}$, $\bar{v}$	velocity components in the $\bar{x}$- and $\bar{y}$-directions;
u, v	nondimensional velocity components in the x- and y-directions;
$\bar{x}$, $\bar{y}$	Cartesian coordinates;
x, y	non-dimensional Cartesian coordinates;
ρ	fluid density;
μ	viscosity;
ν	fluid kinematic viscosity;
α	thermal diffusivity;
β	thermal expansion coefficient;
ψ	dimensional stream function;
σ	Stefan – Boltzman constant;
χ	the mean absorption coefficient.

Subscripts

w	evaluated on the wall;
∞	evaluated at the outer edge of the boundary layer.

Introduction

In a number of applications, the porous medium represents a means to absorb or emit radiant energy that is transferred to or from a fluid. At high temperatures, thermal radiation can significantly affect the heat transfer and the temperature distribution in the boundary layer flow of a participating fluid. The fluid can be assumed to be transparent to radiation, because the dimensions for the radiative transfer among the solid structure elements of porous material are usually much less than the radiative mean free path for scattering or absorption in the fluid [1]. Tong et al. [2] studied the problem of thermal radiation, convection, and conduction in porous media contained in vertical enclosure. Combined radiation and natural convection in a participating medium between concentric cylinders was investigated by Tan and Howell [3]. The problem of radiation, convection, and conduction in porous media contained in two-dimensional vertical cavities was introduced by Bouallou and Sacadura [4]. Forced convection-radiation interaction heat transfer in boundary layers over a flat plate submerged in a porous medium was analyzed by Mansour [5]. Mohammadein and El-Amin [6] studied thermal radiation effects on power-law fluids over a horizontal plate embedded in a porous medium. El-Hakiem and El-Amin [7] studied the effects of thermal radiation effects on non-Darcy natural convection with lateral mass flux. Slimi et al. [8] introduced a transient study of coupled natural convection and radiation in a porous vertical channel using the finite volume method.

The present investigation was undertaken in order to study the effect of thermal radiation on Forchheimer natural convection over a vertical flat plate in a fluid-saturated porous medium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The wall-temperature distribution is assumed to be uniform. The finite element method (FEM) is used for solving the nondimensional governing equations.

1. Analysis

Consider the problem of non-Darcy natural convection-radiation flow and heat transfer over a semi-infinite vertical surface in a fluid-saturated porous medium (see Fig. 1). The governing equations for this problem are given by:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u} + \frac{C\sqrt{K}}{\nu}\bar{u}q = -\frac{K}{\mu}\left(\frac{\partial p}{\partial \bar{x}} + \rho \mathrm{g}\right), \tag{2}$$

$$\bar{v} + \frac{C\sqrt{K}}{\nu}\bar{v}q = -\frac{K}{\mu}\left(\frac{\partial p}{\partial \bar{y}}\right), \tag{3}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) - \frac{1}{(\rho_\infty C_p)_f}\frac{\partial q^r}{\partial \bar{y}}\,, \tag{4}$$

$$\rho = \rho_\infty[1 - \beta(\bar{T} - \bar{T}_\infty)], \tag{5}$$

where

$$q^2 = \bar{u}^2 + \bar{v}^2,$$

along with the boundary conditions

$$\begin{aligned} \bar{y} = 0: &\qquad \bar{v} = 0, \qquad \bar{T}_w = \text{const}; \\ \bar{y} \to \infty: &\qquad \bar{u} = 0, \qquad \bar{T} \to \bar{T}_\infty, \end{aligned} \tag{6}$$

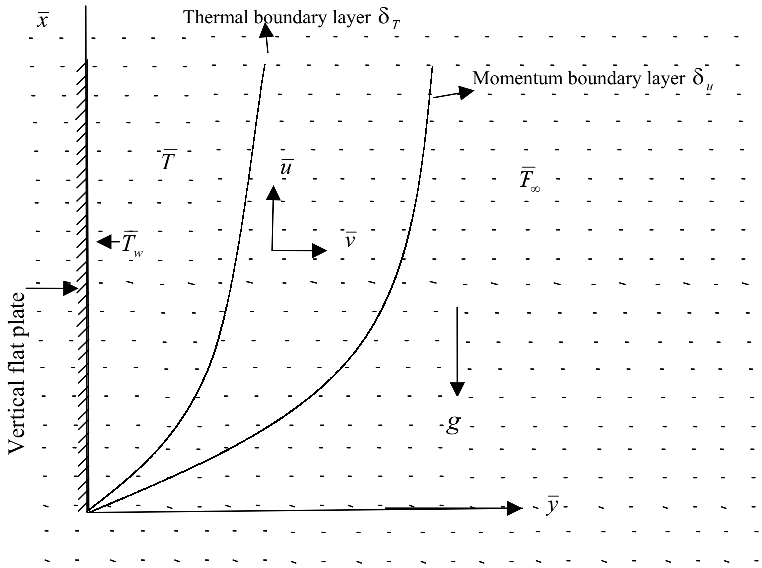

Fig. 1. Physical model and coordinate system.